Chapter 3: Applying Supply and Demand Model

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Learning Objectives

- Elasticities of Demand and Supply.
- Applications of Elasticities: Sales Tax
- Comparative statistics

Elasticity

Elasticity: the percentage change in a dependent variable in response to a given percentage change in an independent variable, ceteris paribus

- Example: ***Price elasticity of demand*** **(ϵ_D)**: is the percentage change in the quantity demanded in response to a percentage change in the price, when all other determinants of demand are kept constant.
- it is crucial in determining by how much equilibrium price and quantity change when the supply or demand curve shift

Price elasticity of demand

- Price elasticity of demand is measured along the demand curve and is given by:

$$\epsilon_D = \frac{\%\Delta Q_d}{\%\Delta P} = \frac{\Delta Q_d/Q_d}{\Delta P/P} = \frac{\Delta Q_d}{\Delta P} \cdot \frac{P}{Q_d} = D'(p) \cdot \frac{P}{Q_D}$$

where $D'(p)$ the slope of the direct demand (or the reciprocal of the slope of the indirect demand) function

- Example: Given the demand function $Q_d = 10 - 2P$, find the point elasticity at $P = 3$ and $P = 4$.
- **Soln**:

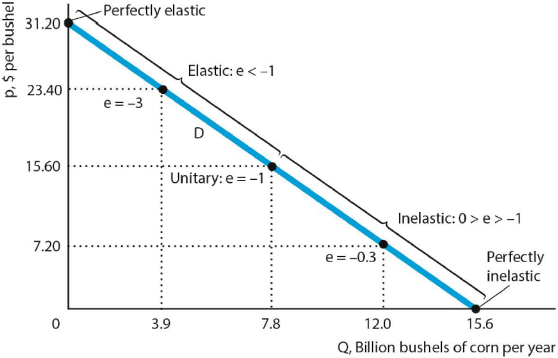
Slope and Price elasticity of demand

- Slope and elasticity are not the same but are related

1. ***Perfectly Inelastic***: slope $= \infty$ (vertical demand curve) and $|\epsilon_D| = 0$
2. ***Inelastic***: $0 < |\epsilon_D| < 1$
3. ***Unit Elastic***: $|\epsilon_D| = 1$
4. ***Elastic***: $1 < |\epsilon_D| > 1$, but less than ∞
5. ***Perfectly elastic***: $|\epsilon_D| = \infty$, slope $=0$ (horizontal curve)

Price elasticity of demand of Linear Demand

With a linear demand curve, the higher The price, the more elastic the demand curve



Determinants of ϵ_D

$|\epsilon_D|$ is bigger

1. when the good has close substitutes: Cereal vs Sunscreen lotion
2. when the good is more narrowly (than broadly) defined: Pizza vs. food in general
3. when the good is a luxury than a necessity: Caribbean cruise vs insulin
4. when the good is already taking a higher share in the consumer's budget: salt vs. rent
5. in the long-run than in the short-run: gasoline demand this week vs this year

Application: ϵ_D and Total Revenue (TR)

1. If $|\epsilon_D| > 1$ (elastic), price and TR inversely related:
   - An increase in price results in decrease in TR
   - A decrease in price results in increase in TR
2. If $|\epsilon_D| < 1$ (inelastic), price and TR directly related
   - An increase in price results in increase in TR
   - A decrease in price results in decrease in TR

ACTIVE LEARNING

1. Pharmacies raise the price of insulin by 10%. Does total expenditure on insulin rise or fall?
2. As a result of a fare war, the price of a luxury cruise falls 20%. Does luxury cruise companies' total revenue rise or fall?

Other Demand Elasticities: Income Elasticity of Demand

$$\epsilon_y = \frac{\%\Delta D}{\%\Delta y} = \frac{\Delta D/D}{\Delta y/y} = \frac{\Delta D}{\Delta y} \cdot \frac{y}{D} = D'(y) \cdot \frac{y}{D}$$

- p is replaced by y and Q_d is replaced by D to indicate that ϵ_y is measured along a shift of a demand curve when p is kept constant
- $\epsilon_y > 0$ for normal good and < 0 for inferior good

Other Demand Elasticities: Cross price Elasticity of Demand

$$\epsilon_{p_0} = \frac{\%\Delta D}{\%\Delta p_0} = \frac{\Delta D/D}{\Delta p_0/p_0} = \frac{\Delta D}{\Delta p_0} \cdot \frac{p_0}{D} = D'(p_0) \cdot \frac{p_0}{D}$$

- ϵ_{P_0} is also measured along a shift of a demand curve when p and other factors are kept constant
- $\epsilon_{P_0} > 0$ for substitute goods and < 0 for complement goods

Numerical Example

- Suppose the demand function for coffee is given by the Cobb-Douglas function of the form:

$$Q = D(p, p_t, y) = 100p^{-.5}p_t^{.6}y^{.7}$$

where p, p_t and y are respectively the price of coffee per cup and p_t price of tea and y aggregate income of consumers, find the cross-price and income-elasticity of coffee at $p = 1, p_t = 2$ and $y = 3$.

Note that:

$$\partial Q/\partial p_t = D'(p_t) = 0.6 \times 100p^{-0.5}p_t^{0.6-1}y^{0.7},$$

$$\partial Q/\partial y = D'(y) = .7 \times 100p^{-0.5}p_t^{0.6}y^{0.7-1}$$

Price Elasticity of Supply

$$\epsilon_S = \frac{\%\Delta Q_S}{\%\Delta P} = \frac{\Delta Q_S/Q_S}{\Delta P/P} = \frac{\Delta Q_S}{\Delta P} \cdot \frac{P}{Q_S} = D'(p) \cdot \frac{P}{Q_S}$$

- It is measured along the supply curve when P changes other determinants of suplly kept constant
- The flatter the supply curve (slope small), the bigger the price elasticity of supply.
- Similar to the demand, ϵ_S is closely related to slope:

1. perfectly inelastic with $\epsilon_S = 0$ (vertical supply)
2. inelastic when ϵ_S is between 0 & 1
3. unite elasticity when $\epsilon_S = 1$
4. elastic when $\epsilon_S > 1$ and
5. perfectly elastic when $\epsilon_S = \infty$ (horizontal supply)

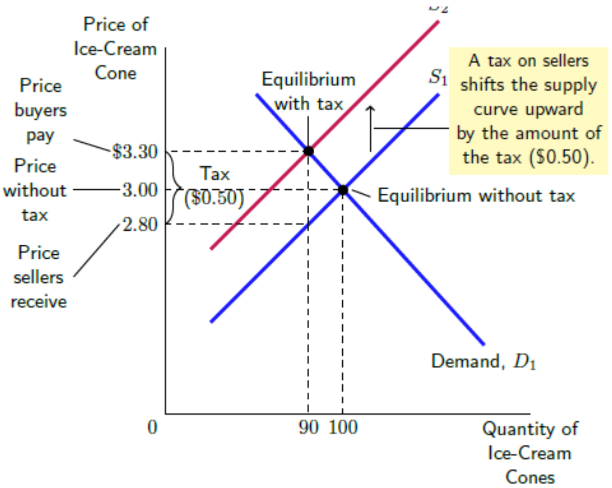
Tax incidence of Specific tax

- One of the most important results in economics is that who writes the check to the govt in collection of tax (statutory burden) doesn't determine who really pays the tax (economic burden)
- The tax burden on the consumers is the increase in the net price paid by consumers due to the tax and the burden on sellers is the decrease in net price received by sellers due to the tax.
- The relative burden depends on the relative price elasticity of demand and supply

Tax incidence of Specific tax: Tax imposed on Sellers

- A per unit tax rate of "τ" shifts the supply curve to the left by vertical distance of "τ". Why?
- or it shifts the supply curve to the left horizontally by $\frac{\Delta Q}{\Delta P} \times \tau$

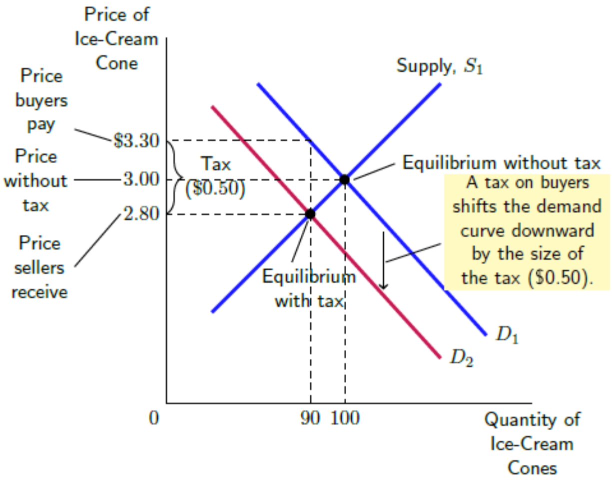
Graphically: Tax imposed on Sellers



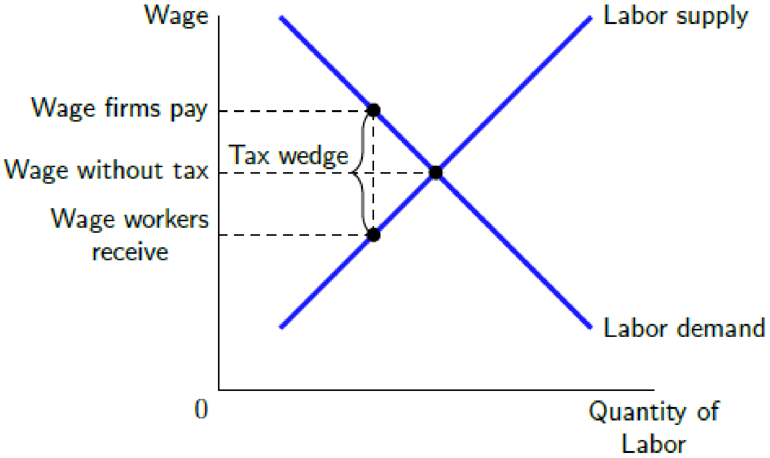
Tax incidence of Specific tax: Tax imposed on Buyers

- A τ per unit tax shifts the demand curve to the left by vertical distance of τ. Why?
- or it shifts the demand curve to the left horizontally by $\frac{\Delta Q}{\Delta P} \times \tau$

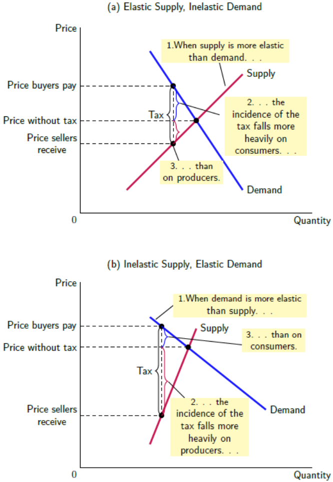
Graphically: Tax imposed on Buyers



Graphically: Tax as a wedge



Graphically: Elasticity and Incidence



Numerical Example

Suppose the following are the demand and supply functions for coffee, respectively, $q_d = 10 - p$ and $q_s = 2p$, which imply initial eqm at $p^* = 3.33$ and $q^* = 6.67$.

Find how much total revenue the govt collects if it imposes a tax of $2 per unit on sellers and what is the share of the tax burden on sellers and buyers.

Tax Incidence & Slopes of Supply and Demdand

- It can be shown that, the tax burden on consumers due to a tax change of Δt can be given by:

$$\Delta p = \frac{S'}{S' - D'} \times \Delta t = \frac{S'}{S' + |D'|} \times \Delta t = \frac{1}{1 + |D'|/S'} \times \Delta t$$

where Δp is the change in price (usually starting from zero) and S' and D' are the slopes (first derivatives) of the 'direct' s

Tax Incidence & Price elasticitieis of Supply and Demdand

Multiplying both $|D'|$ and S' by p/Q, we can express the above in terms of elasticities:

$$\Delta p = \frac{1}{1 + |\epsilon_D|/\epsilon_S} \times \Delta t$$

- The above formula shows that the more elastic the $|\epsilon_D|$ relative to ϵ_S, the smaller the burden on buyers and higher the burden on sellers and vice-versa.
- Who pays the tax on yachts?

Example:

- From our previous numerical example, The per unit tax burden on consumers was $\Delta p \approx \$1.34$ which could have been easily found by:

$$\Delta p = \frac{1}{1 + |D'|/S'} \times dt = \frac{1}{1 + 1/2} \times \$2 \approx \$1.33$$

And then the per unit burden on sellers equal $\$(2 - 1.33) = \$0.66$

Example 2: Working with Elasticities

- When elasticities are readily available, it is easier to use the elasticity version of the above formula.

Suppose the demand function for coffee is

$$Q_D = 100p^{-3}p_t^{.6}y^{.7}$$

and the supply is

$$Q_S = pw^{-1}$$

where w is the wage rate of barrister, and all other variables are as defined before.

Find the per unit tax burden of \$1 on buyers and sellers assuming that $y = p_t = w = 1)$

hence, buyers pay \$.50 and sellers pay $2 - \$.50 = \$1.50$ per unit

Tax incidence with Constant Elasticity Demand and Supply

Suppose the yearly supply of best Ethiopian coffee to U.S. market is given by

$$\ln(Q_S) = 0.25 + .65 \ln(P)$$

and the demand of American's is

$$\ln(Qd) = 2.50 - 0.25 \ln(p) + 0.15 \ln(p_t),$$

where Q is in millions of kgs and P in dollars and P_t is the price of tea per unit, a substitute. Suppose $p_t = \$100$

- Find the equilibrium prices and quantity of coffee before and after the imposition of specific tax of \$2 per kg and the incidence of the tax on buyers and sellers.