

# Chapter 5: Derivation of Demand

Abel Embaye

UofA

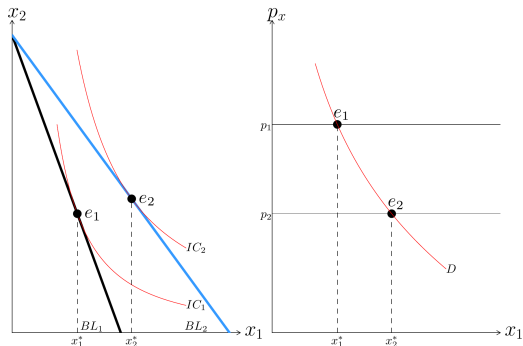
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# Learning Objectives

- Deriving Demand Curves
- How Changes in Income Shift Demand Curves
- Effects of a Price Change
- Cost-of-Living Adjustments
- Deriving Labor Supply Curves

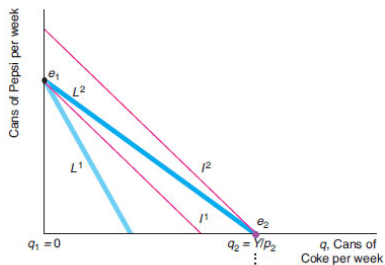
# Price-Consumption Curve (PCC) and Individual's Demand Curve

- **PCC**: A line connecting optimal bundles when the price of one good changes, ceteris paribus.
- The demand curve is derived from the price-consumption curve.

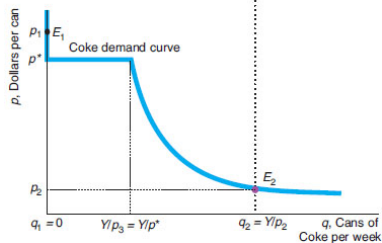


# PCC for perfect substitute

(a) Indifference Curves and Budget Constraints



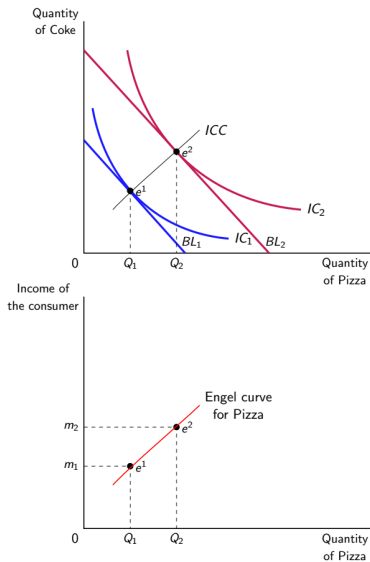
(b) Coke Demand Curve



# Effects of Change in Income

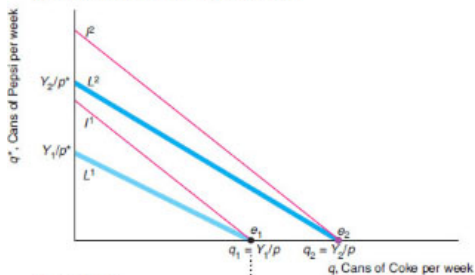
- **Engel curve** is the relationship between the quantity demanded of a single good and income, holding prices constant.
- **Income-consumption curve (ICC)** shows how consumption of both goods changes when income changes, while prices are held constant.

# Income Changes and Engel Curves

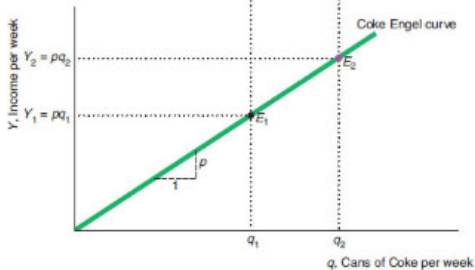


# ICC for Perfect Substitutes

(a) Indifference Curves and Budget Constraints



(b) Engel Curve



# Income Elasticities

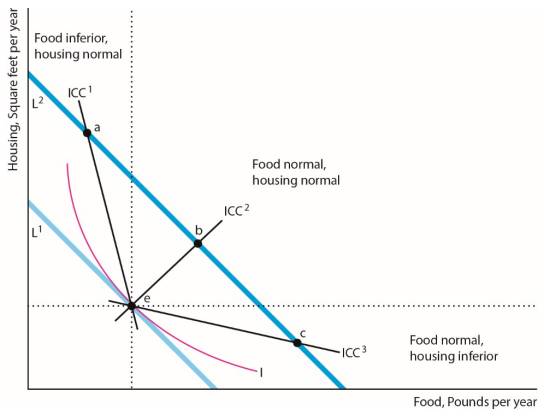
- Recall that,

$$\epsilon_y = \frac{\% \Delta Q}{\% \Delta Y} = \frac{\Delta Q / Q}{\Delta Y / Y} = \frac{\Delta Q / \Delta Y}{Y / Q} = \frac{\Delta Q}{\Delta Y} \cdot \frac{Y}{Q}$$

where  $Y$  stands for income.

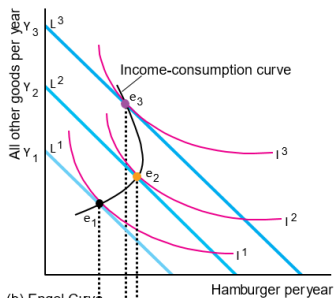
- Example: If a 1% increase in income results in a 3% increase in quantity demanded, the  $\epsilon_y = \frac{3\%}{1\%} = 3$
- For **normal goods**  $\epsilon_y > 0$  and for **inferior** ones  $\epsilon_y < 0$

# ICC and Income Elasticities

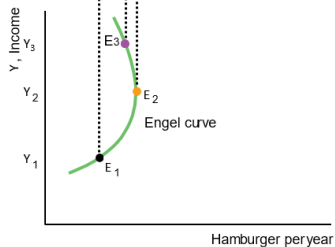


# Application: Fast-food Engel Curve

(a) Indifference Curves and Budget Constraints



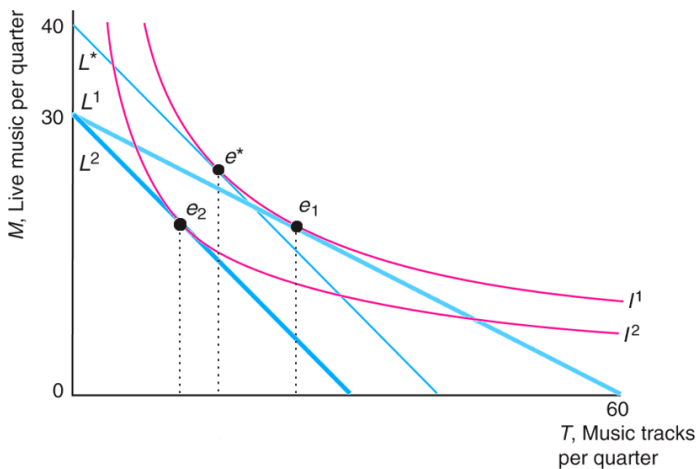
(b) Engel Curve



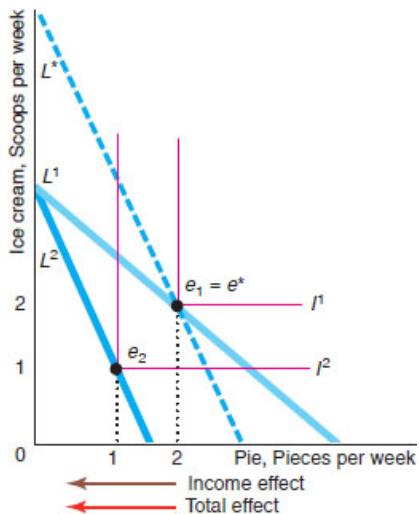
# Decomposition of Price Change: Substitution and Income Effects

- **Substitution effect:** the change in the quantity of a good that a consumer demands when the price of good changes, holding other prices and the consumer's utility constant.
- **Income effect:** the change in quantity demanded of a good income changes, holding prices constant.

# Substitution and Income Effects with price increase for Normal Goods



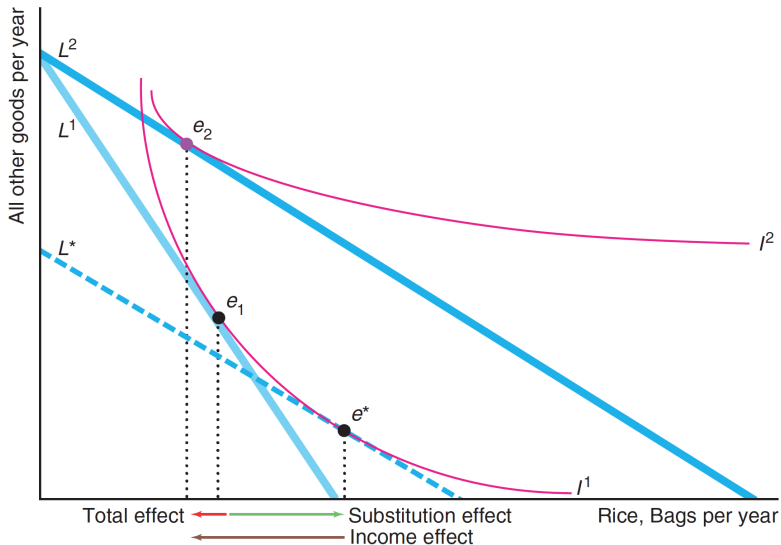
# Substitution and Income Effects with Perfect Complements



# Income and Substitution Effects with an Inferior Good

- If a good is inferior, the income effect and the substitution effect move in opposite directions.
- For most inferior goods, the income effect is smaller than the substitution effect.
- If the income effect more than offsets the substitution effect, we have a **Giffen good**, for which a decrease in its price causes the quantity demanded to fall.

# Income and Substitution Effects with an Inferior Good



# Compensating and Equivalent Variation

- For price increase of a good:
  - **CV**: the amount of money one would have to give a consumer to offset completely the harm from a price increase (change in income required to achieve initial utility at new prices).
  - **EV**: the amount of money one would have to take from a consumer to harm the consumer by as much as the price increase (change in income required to achieve the new utility at initial prices).
- They are two ways of measuring change in consumer welfare due to price change and  $CV \approx EV \approx \text{Change in Consumer Surplus}$

# Compensating and Equivalent Variation: Numerical Example

Tyler consumes pizza ( $z$ ) and coke ( $c$ )'s with:

$$U = cz, \quad p_c = 2, \quad p_z = 2, \quad \text{his income} = \$100$$

- Find the optimal choice of  $z$  and  $c$  for the consumer and the level of  $U$  achieved (label it  $U_1$ ).
- Suppose now price of coke decreases to 1. Find the optimal choice of the good for Tyler and the new  $U$  level (label it  $U_2$ )?
- By how much Tyler's income must be decreased to achieve initial utility level at the new prices (CV)
- By how much Tyler's income must be increased to achieve  $U_2$  with initial prices (EV)

# Price Indexes

- **Inflation:** the increase in the overall price level over time.
  - **nominal price:** the actual price of a good.
  - **real (relative) price:** the price adjusted for inflation.
- How do we adjust for inflation to calculate the real price?

# Price Indexes

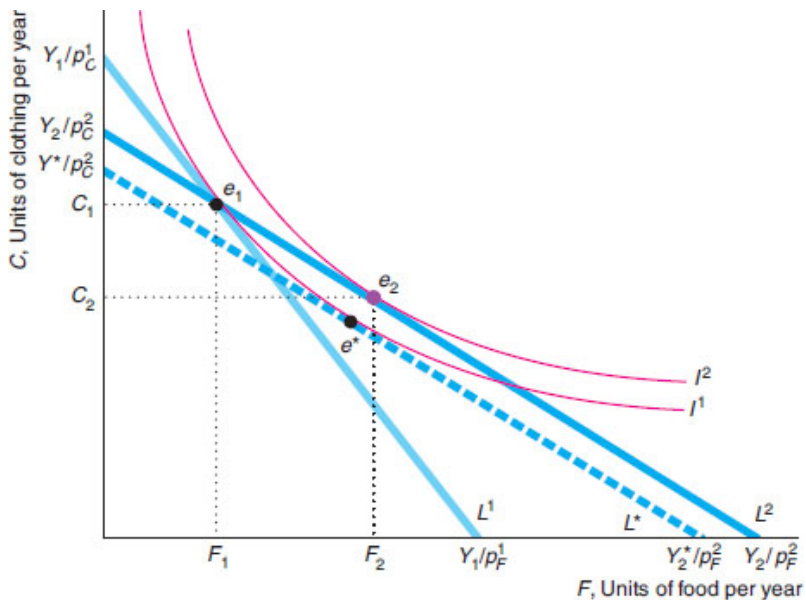
- **Consumer Price Index (CPI)** measures the cost of a standard bundle of goods (called basket) in a given year relative to a based year.
- It is is Used
  1. To make cost of living adjustment (COLA)
  2. calculate the year to rate of inflation
  3. to convert prices to today's dollar:
- Example: If gas price in 1955 was \$.50, in today's dollar this is:

$$\begin{aligned} .50 \times \frac{\text{CPI of 2021}}{\text{CPI of 1955}} &= .50 \times \frac{240.2}{26.8} \\ &= \$4.47 \end{aligned}$$

# Official Inflation is Overstated!

- CPI adjustment tries to restore the initial bundle
- “True COLA adjustment” tries to restore the initial utility level, which usually need smaller change in income!
- **True cost-of-living index** - an inflation index that holds utility constant over time.

# True Cost-of-Living Adjustment



# Cost-of-Living Adjustments table

Assumed Utility:  $U = F \cdot C$

Table 1:

	Pf	Pcl	Income	Food	Clothing	Utility
First Year	4	1	400	50	200	10000.0
Second Year	5	2				
No Adj.			400	40	100	4000.0
CPI Adj.			650	65	162	10562.5
True Adj.			632	63	158	10000.0

# Size of the CPI Substitution Bias

- Income adjustments based on CPI suffer from an upward bias sometimes called the **substitution bias**, it ignores that consumers may substitute toward the relatively inexpensive good when prices change disproportionately.
- A number of studies estimate that the U.S. substitution bias is at least 0.5%.

# Labor-Leisure Choice

- **Leisure** - all time spent not working.
- The number of hours worked per day,  $H$ , equals 24 minus the hours of leisure or nonwork,  $N$ , in a day:

$$H = 24 - N$$

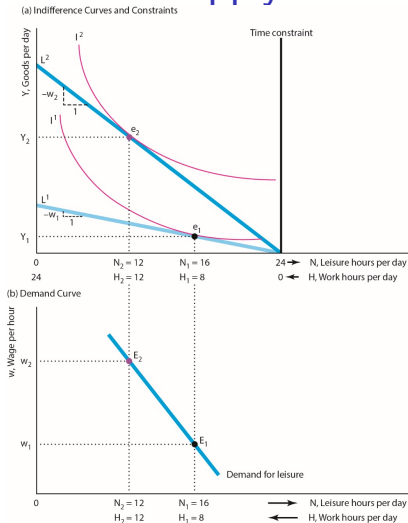
- The price of leisure is forgone earnings: The higher your wage, the more an hour of leisure costs you.

## Labor-Leisure Choice: Example

- Jackie spends her total income,  $Y$ , on various goods .
  - The price of these goods is \$1 per unit.
- Her utility,  $U$ , depends on how many goods and how much leisure she consumes:
- Jackie's earned income equal:  $wH$  from working and  $Y^*$  from other sources such as investment income or government cash assistance

$$Y = WH + Y^*$$

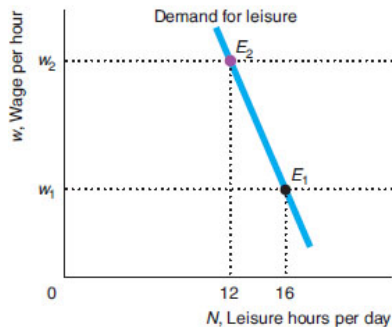
# Demand for Leisure or Supply for labor



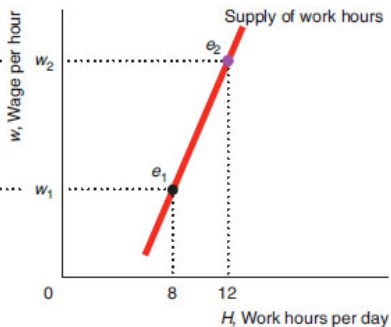
- Budget line,  $L^1$ :  $Y_1 = w_1 H = w_1(24 - N)$
- Budget line,  $L^2$ :  $Y_2 = w_2 H = w_2(24 - N)$

# Supply Curve of Labor

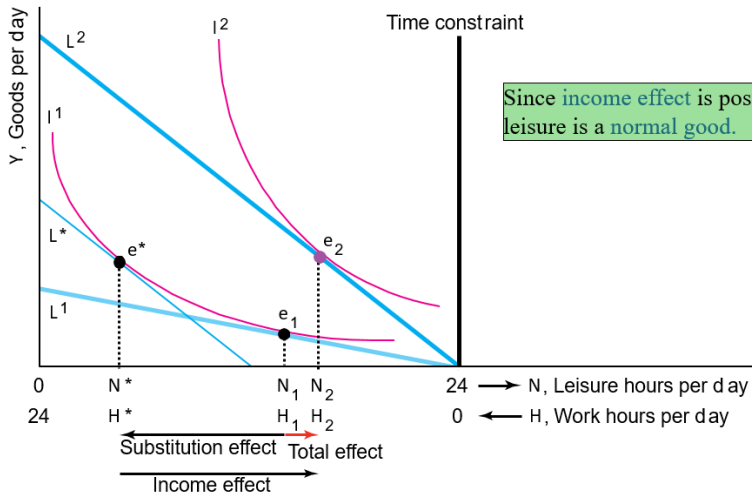
(a) Leisure Demand



(b) Labor Supply

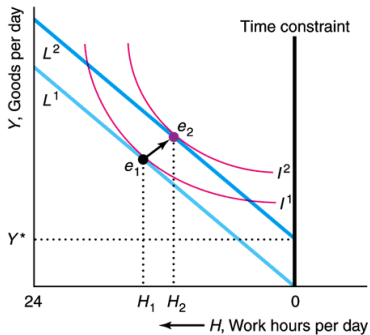


# Income and Substitution Effects of a Wage Change

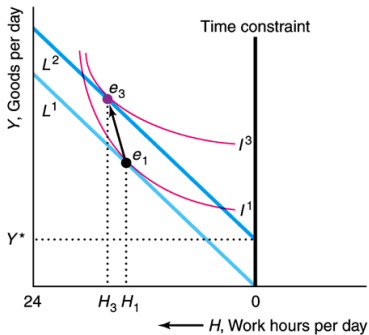


# Two effects of pure income effects

(a) Leisure Normal

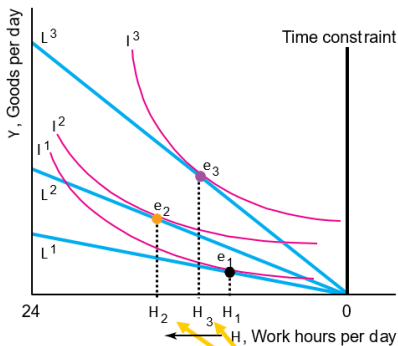


(b) Leisure Inferior

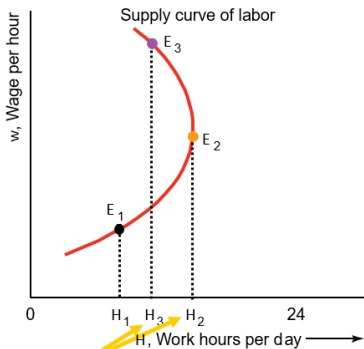


# Backward Bending Labor Supply Curve?

(a) Labor-Leisure Choice

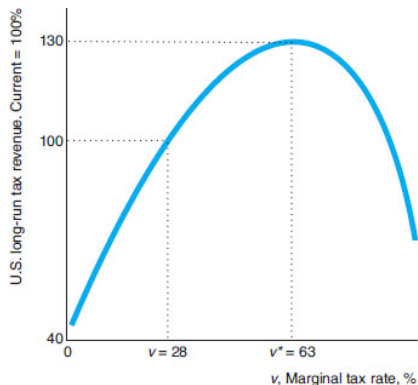


(b) Supply Curve of Labor



but at high wages, an increase in the wage causes the worker to work less....

# Income Tax Rates and Labor Supply and Total tax revenues



- Common error: A cut in the income tax rate must raise government tax revenue
- it depends on tax elasticities of labor supply with respect to tax

# Per-Hour Versus Lump-Sum Child Care Subsidies

