Chapter 6: Production

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Outline

Introduction

- Firm Objective: profit maximization
  - Profit = TR - TC
- In this chapter we study TC and in later chapters TR
- To study cost, study production

Production technology

– Inputs: labor, land, capital (physical), and raw material

– Output: quantity of cars

– ***Production set***: All combinations of inputs and outputs that are technologically feasible

A. Production function: A function describing the boundary of production set

– Mathematically,

$$q = f(x_1, x_2, \ldots)$$

where x = amount of input(s), q = amount of output

Production technology

- Unlike utility functions, two prod. functions do not represent the same technology even if one is a monotone transformation of the other.
- From now on, assume 2 inputs (L, K) and 1 output (Q).

Short-Run Production

- In the short run, the firm's production function is

$$q = f(x_1, \bar{x_2})$$

where q is output, x_1 is the amount of labor, and x_2 is the fixed number of units of capital.

- Example: $q = L^{0.5}K^{.5}$ when k=4

TP, MP and AP

- ***Total product of labor (TP)***: the amount of output (or total product) that can be produced by a given amount of labor.
- ***Marginal product of labor (*** MP_L ***)***: the change in total output, Δq, resulting from using an extra unit of labor, Δx_1, holding other factors (capital) constant:

$$MP_L = \frac{\Delta Q}{\Delta L} = \frac{Q(L + \Delta L, K) - Q(L, K)}{\Delta L}$$

similarly,

$$MP_K = \frac{\Delta Q}{\Delta K} = \frac{Q(L, K + \Delta K) - Q(L, K)}{\Delta K}$$

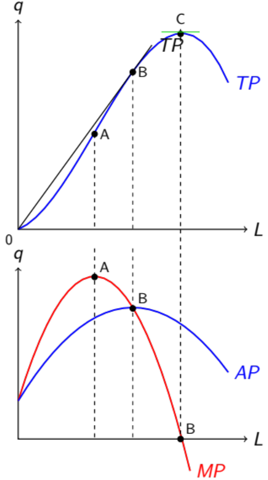
TP, MP and AP

- ***Average product of labor (APL)***: the ratio of output, q, to the number of workers, L, used to produce that output:

$$AP_i = q/x_i$$

  - Example: Find TP, AP and MP for $q = 2L + K$ at L= 100 when K is fixed at k=2?

TP, MP and AP of typical production function



- ***Diminishing returns***: Notion that *MP* of input declines eventually, shown that TP increases at decreasing rate or declining portion of MP_L curve.

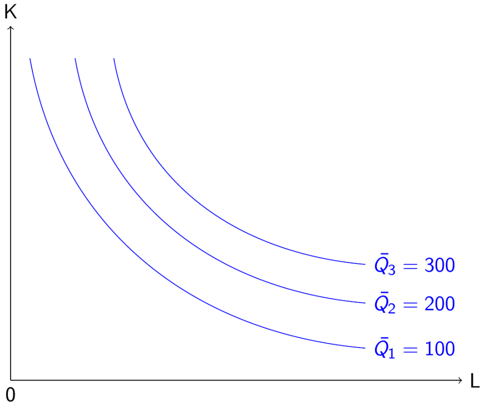
Long run Production

- In the long run, all inputs are variable
- **Isoquant:** Set of all possible combinations of input 1 and 2 that yields the same level of output, that is

$$(x_1, x_2) \text{ such that } f(x_1, x_2) = Q, \text{ for a given Q}$$

Example: find the isoquant with Q=100 for the production function $q = \sqrt{LK}$

Isoquant Map



Properties of Isoquants

1. Isoquants slope downward.
2. The farther an isoquant is from the origin, the greater the level of output.
3. Isoquants do not cross.

Slope of Isoquant: Margianl Rate of Technical transformation (MRTS)

- MRTS is the slope of the isoquant i.e. the ratio of change in K over change in L when Q is constant:

$$MRTS_{L,K} = \frac{\Delta K}{\Delta L}$$

- There is close relationship between MRTS, MPL and MPK:

$$MRTS_{L,K} = \frac{\Delta K}{\Delta L} = \frac{\Delta K / \Delta Q}{\Delta L / \Delta Q} = -\frac{1/MP_K}{1/MP_L} = -\frac{MP_L}{MP_K}$$

$MRTS = -MP_L/MP_K$

Exercise:

Given $q = 2L^{0.7}K^{0.3}$ which implies that
$MP_L = 2 \times 0.7L^{-0.3}K^{0.3}$ and $MP_K = 2(.3)L^{0.7}K^{-0.7}$,

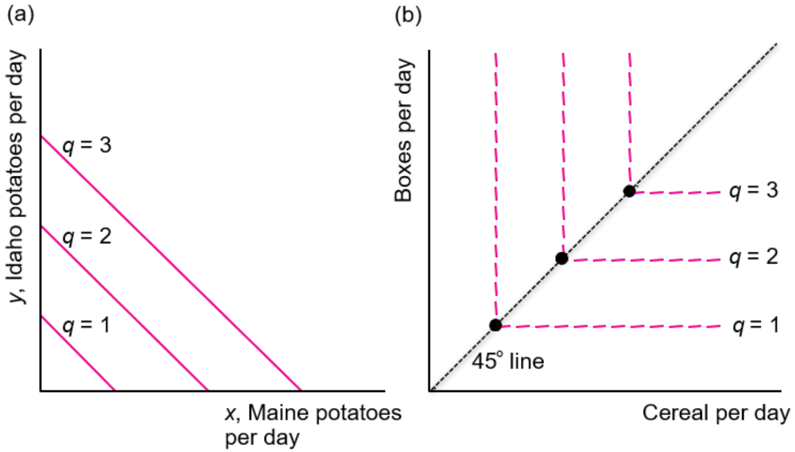
- find the equation for the isoquant at Q=100 and show that $MRTS = -MP_L/MP_K$

Perfect and Imperfect Substitutes of Inputs

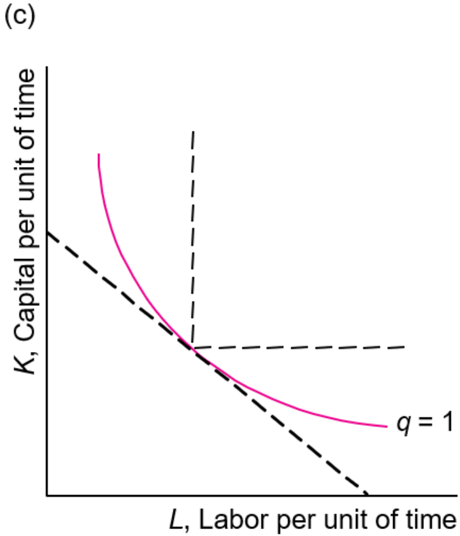
Like in consumer theory, inputs can be perfect substitutes, complement, etc

- If MRTS is constant along Isoquant, the inputs are ***perfect substitutes***: e.g., $q = L + K$
- if inputs are ***perfect complements***, we have ***fixed proportion***, sometimes called ***Leontief Isoquants***
- Most isoquants are in between the two: ***imperfect substitutes***

Perfect Substitutes and Perfect Complements in Inputs



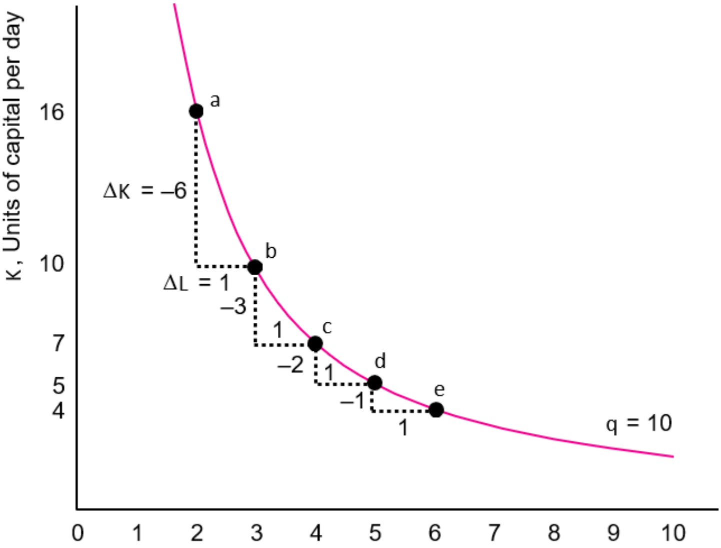
The Imperfect substitute case as intermediate case



Desirable properties of technology

1. **Monotonic:** f is increasing in L and K
2. **Convex Isoquants:** Given two bundles of inputs
   $x_0 \quad \text{and} \quad x_1,$
   $f(\lambda x + (1-\lambda)x_0) \geq \lambda f(x) + (1-\lambda)f(x_0) \quad for \quad \lambda \in [0,1]$
   i.e., the weighted average of two combinations of inputs gives more output than each of the two input combinations
3. **Diminishing Returns:** for each input x_i, MP_{x_i} is decreasing with x_i
4. **Decreasing MTRS:** $MRTS$ is decreasing with increased use of x_i along the Isoquant

Diminishing MRTS



Returns to scale: How much output increases as all inputs are scaled up Proportionally?

$$f(tL, tK) \begin{pmatrix} > \\ = \\ < \end{pmatrix} tf(L, K) \iff \begin{pmatrix} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{pmatrix} \text{return to scale}$$

Return to Scale: Exercise

Show whether the following production functions are IRS, CRS or DRS

a. $f(x_1, x_2) = x_1 x_2$

b. $f(x_1, x_2) = min\{x_1, x_2\}$

c. $f(x_1, x_2) = x_1 + x_2$

d. Under what conditions would $q = AL^{\alpha} K^{\beta}$ would be CRS, IRS, DRS?